The Power of Connectivity: Identity Preserving Transformations on Visual Streams in the Spike Domain
List of Supplementary Materials *

Aurel A. Lazar
Department of Electrical Engineering
Columbia University, New York, NY 10027
aurel@ee.columbia.edu

Eftychios A. Pnevmatikakis
Department of Statistics and Center for Theoretical Neuroscience
Columbia University, New York, NY 10027
eftychios@stat.columbia.edu

Yiyin Zhou
Department of Electrical Engineering
Columbia University, New York, NY 10027
yiyin@ee.columbia.edu

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The supplementary materials consists of a total of six videos and two figures.

1. **video1.mov:**
   Recovery and Rotations on the Log-Polar Grid. The video consists of 4 parts. Starting from left to right, they are: **(A)** Original visual stream. **(B)** Recovery with a null setting switching matrix. **(C)** Recovery with a switching matrix setting realizing a rotation of 45 degrees counter-clockwise. **(D)** Recovery with a switching matrix setting realizing a rotation of 171 degrees clockwise. SSIM index for (B-D) is, respectively, $0.97, 0.97, 0.97$.

   Difference of Gaussian receptive fields were used. The parameters of the Log-Polar grid are the same as in Section 3.1:

   - $\alpha_0 = 2, m \in \{-3, -2, -1, 0, 1\}$,
   - $\theta_0 = 2\pi/L, L = 120, l \in \{0, 1, \cdots, 119\}, \omega_0 = 2\pi, N = 1$,
   - $b_0 = 0.8$.

2. **video2.mov:**
   Dilations and Rotations on the Log-Polar Grid. The video consists of 4 parts. Starting from left to right, they are: **(A)** Original visual stream. **(B)** Recovery with a switching matrix setting realizing a dilation by a factor of 2. The result was multiplied by 2 before being displayed. The region of reconstruction corresponds to the region inside the dashed circle in the original visual stream. **(C)** Recovery with a switching matrix setting realizing a dilation by $1/2$. The result was multiplied by $1/2$ before being displayed. The region of reconstruction corresponds to the region inside the solid circle in the original visual stream. **(D)** Recovery with a switching matrix setting realizing a simultaneous dilation by $1/2$ and rotation by 63 degree counter-clockwise. The result was multiplied by $1/2$ before being displayed. The region of reconstruction corresponds to the region inside the solid circle in the original visual stream. SSIM index for (B-D) is, respectively, $0.97, 0.90, 0.91$.

   The same receptive fields and grid parameters as in video1 above were used.

3. **video3.mov:**
   Recovery and Translations on the Cartesian Grid. The video consists of 4 parts. Starting from left to right, they are: **(A)** Original visual stream. **(B)** Recovery with a null setting switching matrix. **(C)** Recovery with a switching matrix setting realizing a translation by 2 units to the left and by

2 units downwards. **(D)** Recovery with a switching matrix setting realizing a translation by 4 units upwards. SSIM index for (B-D) is, respectively, $0.97, 0.96, 0.96$.

Gabor receptive fields were used. The parameters of the Cartesian grid are the same as in Section 3.2:

- $\alpha_0 = 2, m \in \{0, 1, 2, 3\}$
- $b_0 = 2$,
- $\omega_0 = 2\pi/N, N = 8, n \in \{0, 1, 2, 3, 4, 5, 6, 7\}$,

4. **video4.mov:**
Dilations and Translations on the Cartesian Grid. The video consists of 4 parts. Starting from left to right, they are: **(A)** Original visual stream. **(B)** Recovery with a switching matrix setting realizing a dilation by a factor of 2. The result was multiplied by 2 before being displayed. The region of reconstruction corresponds to the region inside the dashed rectangular frame in the original visual stream. **(C)** Recovery with a switching matrix setting realizing a dilation by a factor of $1/2$. The result was multiplied by $1/2$ before being displayed. The region of reconstruction corresponds to the region inside the solid rectangular frame of the original visual stream. **(D)** Recovery with a switching matrix setting realizing a simultaneous dilation by a factor of $1/2$ and a translation by 1 unit to the right. The result was multiplied by $1/2$ before being displayed. SSIM index for (B-D) is, respectively, $0.90, 0.90, 0.90$.

The same receptive fields and grid parameters as in video2 above were used.

5. **video5.mov:**
Approximate Rotations on the Cartesian Grid. The video consists of 4 parts. Starting from left to right, they are: **(A)** Original visual stream. **(B)** Recovery with a switching matrix setting realizing an approximate rotation by 46 degrees clockwise. **(C)** Recovery with a switching matrix setting realizing an approximate rotation by 92 degrees clockwise. **(D)** Recovery with a switching matrix setting realizing an approximate rotation by 138 degrees clockwise. SSIM index for (B-D) is, respectively, $0.90, 0.90, 0.90$.

Difference of Gaussian receptive fields were used. The parameters of the Cartesian grid are the same as in Section 3.3:

- $\alpha_0 = 2, m \in \{0, 1, 2, 3\}$,
- $\omega_0 = 2\pi, N = 1$,
- $b_0 = 0.8$.

6. **video6.mov:**
   Approximate Translations on the Log-Polar Grid. The video consists of 4 parts. Starting from left to right, they are: **(A)** Original visual stream. **(B)** Recovery with a switching matrix setting realizing an approximate translation by 0.5 units to the right. **(C)** Recovery with a switching matrix setting realizing an approximate translation by 1 unit to the right. **(D)** Recovery with a switching matrix setting realizing an approximate translation by 1.5 units to the right. SSIM index for (B-D) is, respectively, $0.97, 0.94, 0.94$.

   The same receptive fields and grid parameters as in video1 above were used.

7. **Figure S1:**
   Figure S1 illustrates an example of discretization of the SIM(2) group to Log-Polar Grid, as defined by (15) in Section 2.2.3, where $N = 8, L = 30, \alpha_0 = 2, b_0 = 0.8$, and the spatial grid of three levels of $m \in \{0, -1, -2\}$ is depicted in each of the subfigures, corresponding to three different dilation levels (dilation parameter of the group elements). Each element in the grid is represented by a small oriented line segment. The spatial position of the center of a line segment corresponds to the translation parameter of group element, and the angle between a line segment and x axis corresponds to the rotation parameter of the group element. The color of each line segment corresponds to a particular value of n in (15) modulo 4, where blue, magenta, red and black correspond to $n = 0, 1, 2, 3$, respectively. Note that at the origin, only one of the L elements are shown for clarity.

8. **Figure S2:**
   Figure S2 illustrates an example of discretization of the SIM(2) group to Cartesian Grid, as defined by (22) in Section 2.2.4, where $N = 8, \alpha_0 = 2, b_0 = 0.8$, and the spatial grid of three values of $m \in \{0, 1, 2\}$ is depicted in each of the subfigures, corresponding to three different dilation levels (dilation parameter of the group elements). Each element in the grid is represented by a small oriented line segment. The spatial position of the center of a line segment corresponds to the translation parameter of group element, and the angle between a line segment and x axis corresponds to the rotation parameter of the group element. The color of each line segment

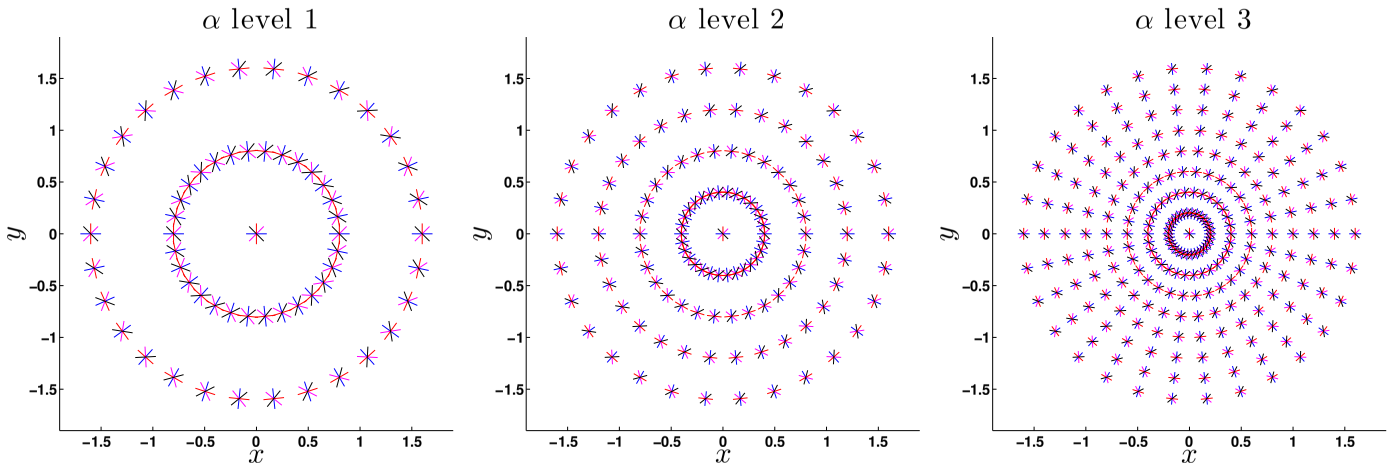

Figure 1: An example of Log-Polar Grid.

corresponds to a particular value of n (22) modulo 4, where blue, magenta, red and black correspond to $n = 0, 1, 2, 3$, respectively.

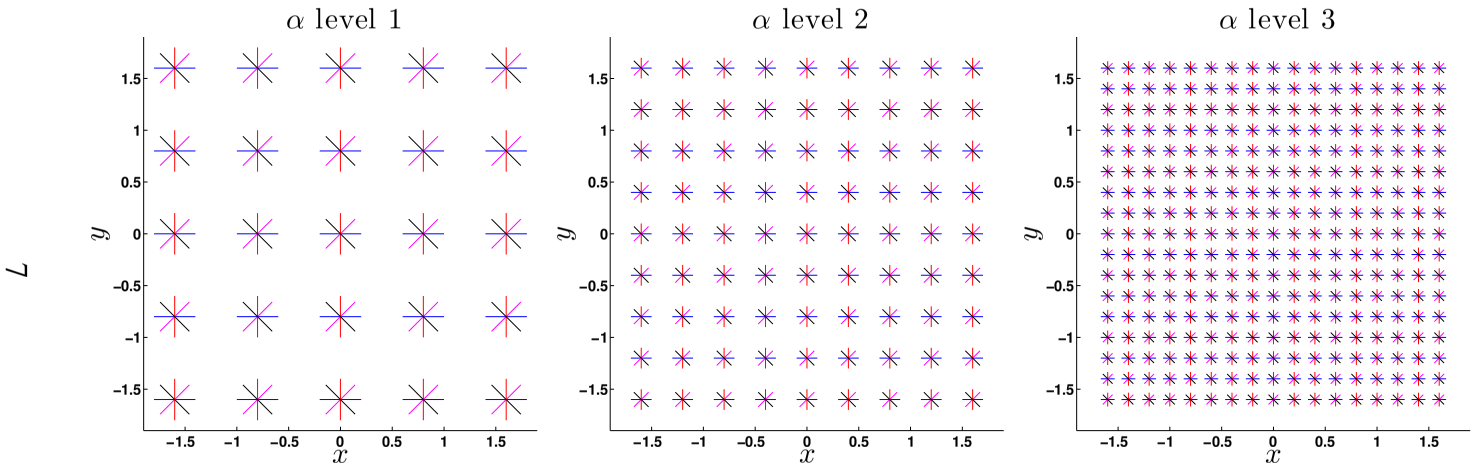

Figure 2: An example of Cartesian grid.